MATHEMATICAL MODEL OF FOUR-STROKE COMBUSTION ENGINE WORKING PROCESS

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Abstract

By defining the fluid's thermodynamic properties, the cycle can be simplified using various assumptions. A lot of models of the combustion engine process were developed that including the thermodynamics, turbulence, and chemical kinetics to predict thermodynamic parameters of the engines.

Mathematical model the working process occurring in the cylinder of the four-stroke piston-combustion engine is an object of the paper. The following assumptions consisting in that thermodynamics system is an open thermodynamical system, in accepted model were taken into account.

Generalised mathematical model of the working process in the cylinder of the piston-four-stroke combustion engine was worked out. Dependences describing instantaneous volume of the working charge in the cylinder, equation of the balance of the of the working charge quantity in the cylinder and state equation of the working charge in the cylinder were considered at formulating of assumptions. Values of coefficients for each component processes of working cycle with taking into consideration of the dependence concerning the compression process , the combustion process, and expansion process after finishing of the combustion process were an object of worked out mathematical model. For nonstationary processes formulating working cycle of the engine realized during openings of valves (the open system), the quantity and composition of the working charge in the cylinder, its specific heat and temperature are variable. The temperature difference between the working charge and walls temperature of bordering the space of the cylinder was taken into account, too. Generalized mathematical model of the working process in the cylinder of the piston- four-stroke combustion engine applying for the theoretical analysis of the working process in the combustion engine, as well as for working out of experimental results was elaborated.

Keywords: *combustion engine, combustion engine thermodynamics, working process, mathematical modelling,*

1. Introduction

When developing the ideal cycles the designer have to determine which model to use for the working fluid within the cylinder. By defining the fluid's thermodynamic properties, the cycle can be simplified using various assumptions. A lot of models of the combustion engine process were developed that including the thermodynamics, turbulence, and chemical kinetics to predict thermodynamic parameters of the engines. For example the conversion of chemical energy into thermal energy by combustion as a separate heat-addition process using a one-zone model based on the first law of thermodynamics where the cylinder-gas composition is modelled as being constant during the combustion cycle. The one-zone model can be extended to a two-zone model in order to compute a more accurate distribution of combustion reactants and products during the combustion cycle of an engine. Many computational reacting flow dynamics models were developed into readily available software packages to facilitate the analysis of combustion in generalized modern internal combustion engines. These models incorporate the relevant subset of thermochemical reactions that occur in the combustion chamber to predict a time and space-resolved distribution of combustion constituents during the combustion cycle. The fluid in the cylinder is constantly undergoing a change in mixture composition. Once the composition is determined, the partial mixture properties can be summed and appropriately weighted in accordance with their mass or mole fractions. This method very simply allows thermodynamic property calculation for mixtures containing residual gas, exhaust gas recirculation, unburned gaseous fuel, air.

This is to be expected since the goals of a control-oriented model are quite different from those of a single cycle combustion model. It is not computationally reasonable to attempt control of extremely fast chemical kinetics involving hundreds of reactions. In order to evaluate single cylinder dynamics and cylinder-to-cylinder variation of internal combustion engines, detail of the fast dynamics of discrete engine events have to be modelled. In order to evaluate single cylinder dynamics and cylinder-to-cylinder variation of internal combustion engines, detail of the fast dynamics of discrete engine events have to be modelled. The one-dimensional quasisteady flow model is used to model flow through both the intake and exhaust valves during the gas exchange processes coefficient, valve area, gas properties, and pressure differential across each orifice. The values for both valve lift and discharge coefficients can be specified or predicted. The values for cylinder pressure are updated by solving the system of state differential equations through the cycle. The combustion process in a homogenous charge compression ignition engine exhibits no fundamental mixing or entrainment, which normally control the combustion event for direct-injection as well as spark ignition engines. As a result the rate of heat release is solely driven by the chemical kinetic reaction rates.

2. Mathematical model of the work process of the four-stroke piston engine

Fig. 1 presents the diagram of the engine cylinder and the elementary processes proceeding in it and in the systems connected with it.

The thermodynamic system presented in Fig. 1 is an open thermodynamic system, therefore the equation of the first principle of thermodynamics for it has the form:

$$\delta E = dU + pdV , \tag{1}$$

where:

δE - elementary quantity of energy delivered to the system,
dU - change of the internal energy of the system,
pdV - elementary work of the working charge in the cylinder.

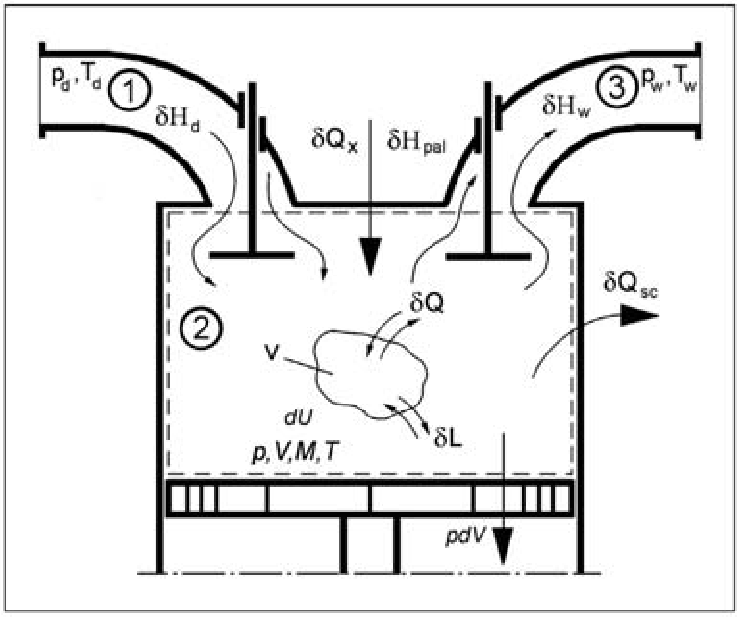

Fig. 1. Elementary processes in the engine cylinder and in the systems connected with it

During an elementary time interval dt, the quantity δM_{12} of moles of the working charge is delivered to the engine cylinder if $p_d > p$, or δM_{21} moles of the working charge flow out of it if $p > p_d$. In result of the flow of the working charge through the inlet valves, the external exchange of energy is:

$$\delta H_d = \delta H_{12} - \delta H_{21} = \delta M_{12}\bar{c}_{psr}T_d - \delta M_{21}\bar{c}_{psr}T \ , \tag{2}$$

where:

$\bar{c}_{psr}$ - molar specific heat of the working charge at constant pressure,

T_d and T – temperatures, in the inlet system, and in the engine cylinder, respectively.

During the time interval dt indicated above, δM_{23} moles of the working charge flow out of the cylinder through the exhaust valves if $p > p_w$, or δM_{32} moles of the exhaust gas flow into it from the exhaust system if $p_w > p$.

The external change of energy for the exhaust valves is:

$$\delta H_w = \delta H_{23} - \delta H_{32} = \delta M_{23}\bar{c}_{psr}T - \delta M_{32}\bar{c}_{psr}T_w \ , \tag{3}$$

where:

Tw - temperature of the exhaust gas in the exhaust system.

During the time dt, delivered to the cylinder are δMpal moles of fuel having the enthalpy value:

$$\delta H_{pal} = \delta M_{pal}\bar{c}_{psr}T_{pal} \ , \tag{4}$$

where:

M_{pal} - quantity of moles of fuel,

T_{pal} - temperature of fuel.

During the chemical combustion reaction, the following quantity of heat is emitted in the engine cylinder during the elementary time interval:

$$\delta Q_x = g_c W_u \frac{dx}{d\alpha} d\alpha \ , \tag{5}$$

where:

g_c - fuel dose per one work cycle of the engine,

W_u - calorific value of the fuel,

x - relative quantity of heat emitted in the cylinder,

$d\alpha$ - change of the engine crank angle during the time dt, which is equal to $d\alpha = 6ndt$.

Because there is a difference between the temperature of the working charge and the temperature of the walls limiting the cylinder space, the quantity of heat exchanged during the elementary time interval is:

$$\delta Q_{sc} = \alpha_g F(T - T_{sc})dt, \tag{6}$$

where:

α_g - coefficient of taking over the heat,

F - contact surface area of the working charge with the walls of the cylinder space.

During the piston movement, the working charge performs the elementary work:

$$\delta L = pdV . \tag{7}$$

The change of internal energy of the working charge during the elementary time interval can be calculated from the formula:

$$dU = d(M\bar{c}_{vsr}T) \tag{8}$$

and the elementary quantity of energy delivered to the cylinder is:

$$\delta E = \delta H_{pal} + \delta Q_x + \delta H_d - \delta H_w - \delta Q_{sc} . \tag{9}$$

Substituting relationships (23) and (24) to equation (16), we obtain the equation of the first principle of thermodynamics in a general form for the working charge realising the work cycle of a piston engine:

$$\delta H_{pal} + \delta Q_x + \delta H_d - \delta H_w - \delta Q_{sc} = d(M\bar{c}_{vsr}T) + pdV . \tag{10}$$

We will obtain a generalised mathematical model of the work process in the cylinder of a four-stroke piston engine, if we complete equation (25) with the following relationships and equations:
- relationship describing the momentary volume of the working charge in the cylinder:

$$V = \frac{V_s}{\varepsilon - 1}\left[1 + (\varepsilon - 1)\frac{\sigma}{2}\right], \tag{11}$$

- equation of the balance of the quantity of the working charge in the cylinder:

$$M_{cz} = \int(\delta M_d - \delta M_w), \tag{12}$$

- equation of the state of the working charge in the cylinder:

$$pV = \bar{R}MT . \tag{13}$$

Instantaneous volume of the working charge in the cylinder:

$$V = \frac{V_s}{\varepsilon - 1}\left[1 + (\varepsilon - 1)\frac{\sigma}{2}\right], \tag{14}$$

where:

V_s - piston displacement volume,

ε - compression ratio,

$\sigma = (1-\cos\alpha) + \frac{\lambda_k}{4}(1-\cos 2\alpha)$ - relative piston displacement,

$\lambda_k = \frac{R}{L}$ - connecting rod coefficient, being the ratio of the crankthrow to the connecting rod length.

3. Specific heat of the working cycle

Relationships describing the specific heat of the working charge, e.g. treated as a semi-ideal gas, in linear dependence on the temperature, are utilised in mathematical modelling of the processes in the engine cylinder:

$$\bar{c}_v = a_v + bT \tag{15}$$

or

$$\bar{c}_p = a_p + bT, \tag{16}$$

where:

$a_p = a_v + \bar{R}$,

$\bar{R}$ - universal gas constant.

In the above relationships of specific heats, usually it is assumed that:

for air: $a_v = a_\lambda = 19.81 \frac{J}{mol \cdot K}$, $b = b_\lambda = 419 \cdot 10^{-5} \frac{J}{mol \cdot K^2}$,

for exhaust gas: $a_v = a_\gamma = 19.86 + \frac{1.634}{\lambda} \frac{J}{mol \cdot K}$, $b_v = b_\gamma = \left(427.3 + \frac{184.3}{\lambda}\right) \cdot 10^{-5} \frac{J}{mol \cdot K^2}$.

The values of coefficients a_v and b of equation (29) for individual component processes of the work cycle are calculated from the relationships:
- for the compression process:

$$a_v = a_{spr} = \frac{a_\lambda + \gamma a_\gamma}{1+\gamma}, \tag{17}$$

$$b = b_{spr} = \frac{b_\lambda + \gamma b_\gamma}{1+\gamma}, \tag{18}$$

where:

γ - coefficient of the exhaust gas residue,
- for the combustion process:

$$a_v = (1-x)a_{spr} + x \cdot a_\gamma, \tag{19}$$

$$b = (1-x)b_{spr} + x \cdot b_\gamma, \tag{20}$$

- for the expansion process, after the combustion process is completed:

$$a = a_\gamma, \quad b = b_\gamma. \tag{21}$$

4. Quantity of the working charge

Due to the non-stationary character of the processes constituting the engine work cycle, realised during the time of opening the valves (open system), the quantity and composition of the working charge in the cylinder and its specific heat and temperature are variable. Knowledge of reliable values of those quantities has critical influence on the reliability of the results, obtained both in result of calculations conducted according to the proposed mathematical model of the cycle, and in result of analysis of the results obtained experimentally.

The quantity of the working charge in the cylinder during the compression process can be calculated from the relationship:

$$M = \eta_v M_s (1+\gamma) = \lambda g_c M_o (1+\gamma) = \text{const}, \tag{22}$$

where:

η_v - value of the cylinder filling ratio,

$M_s = \frac{p_d V_s}{\bar{R} T_d}$ - theoretical quantity of moles of the working charge that can be contained in the piston displacement volume V_s at the pressure p_d and the temperature T_d prevailing in the inlet system, before the engine inlet valves.

The quantity of the working charge in the cylinder, during the period from the beginning of the combustion process to the moment of its end, is calculated from the formula:

$$M = M_{ps}\left(1 + \int_{\alpha_{ps}}^{\alpha} \frac{\beta_o - 1}{1+\gamma}\dot{x}\,d\alpha\right), \tag{23}$$

where:

M_{ps} - quantity of the working charge at the moment of the beginning of the combustion process,

β_o - chemical coefficient of the molar changes during the combustion process,

γ - coefficient of the exhaust gas residue,

$\dot{x}$ - velocity of the relative quantity of heat emitted during the combustion.

The quantity of the working charge during the expansion process, counting from the end of the combustion process to the moment of opening the exhaust valve, is:

$$M = \beta M_{ps}, \tag{24}$$

where:

$\beta = 1 + \frac{\beta_o - 1}{1+\gamma}$ - effective coefficient of the molar changes,

$$\beta_o = 1 + \frac{\frac{H}{4} + \frac{O}{32}}{\lambda \cdot M_o}.$$

During the process of exchange of the working charge, occurring from the moment of opening the exhaust valve to the closing of the inlet valve, we calculate:

$$M = M_{ow} + \int_{\alpha_{ow}}^{\alpha}\left(\frac{\delta M_{12}}{d\alpha} - \frac{\delta M_{21}}{d\alpha}\right)d\alpha - \int_{\alpha_{ow}}^{\alpha}\left(\frac{\delta M_{23}}{d\alpha} - \frac{\delta M_{32}}{d\alpha}\right)d\alpha, \tag{25}$$

where:

M_{ow} - quantity of the working charge in the cylinder at the moment of opening the exhaust valve,

α_{ow} -value of engine crank angle when the exhaust valve is opening.

The value of throughput of the working charge flow in function of the crank angle is calculated from the relationship:

$$\frac{\delta M}{d\alpha} = \frac{(\mu_z f_z) w_z \rho_{zsr}}{\mu \cdot 6n}, \tag{26}$$

where:

$\mu_z f_z$ - effective coefficient of the flow throughput through the considered passage section,

w_z - theoretical velocity of flow,

ρ_{zsr} - mean density of the working charge flowing through the passage section,

μ - molar mass of the flowing charge,

n - rotational velocity of the engine crankshaft.

The velocity of flow of the working charge through the inlet valves:

a) If $p < p_d$ and $\mu_d f_d \neq 0$, then

$$w_{12} = \sqrt{2RT_d \frac{\kappa_d}{\kappa_d - 1}\left[1 - \left(\frac{p}{p_d}\right)^{\frac{\kappa_d - 1}{\kappa_d}}\right]}, \tag{27}$$

$$\rho_{zsr} = \frac{p}{RT_{dsr}}, \quad T_{dsr} = T_d \left(\frac{p}{p_d}\right)^{\frac{\kappa_d - 1}{\kappa_d}}. \tag{28}$$

b) If $p > p_d$ and $\mu_d f_d \neq 0$, then

$$\frac{p}{p_d} \geq \beta = \left(\frac{2}{\kappa + 1}\right)^{\frac{\kappa}{\kappa - 1}}, \tag{29}$$

$$w_{21} = \sqrt{2RT \frac{\kappa}{\kappa - 1}\left[1 - \left(\frac{p_d}{p}\right)^{\frac{\kappa - 1}{\kappa}}\right]}, \tag{30}$$

$$\rho_{zsr} = \frac{p_d}{RT_{dsr}}, \quad T_{dsr} = T_d \left(\frac{p_d}{p}\right)^{\frac{\kappa - 1}{\kappa}}. \tag{31}$$

The velocities and other parameters of the working charge flowing through the exhaust valves:

a) If $\frac{p_w}{p} \leq \beta = \left(\frac{2}{\kappa + 1}\right)^{\frac{\kappa}{\kappa - 1}}$ the critical velocity of flow and $\mu_w f_w \neq 0$, then

$$w_{23} = \sqrt{2RT \frac{\kappa}{\kappa + 1}}, \quad \rho_{zsr} = \frac{p}{RT} \cdot \beta^{\frac{1}{\kappa}}. \tag{32}$$

b) If $\frac{p_w}{p} > \beta = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}$ and $\mu_w f_w \neq 0$, then

$$w_{23} = \sqrt{2RT\,\frac{\kappa}{\kappa-1}\left[1-\left(\frac{p_w}{p}\right)^{\frac{\kappa-1}{\kappa}}\right]}, \tag{33}$$

$$\rho_{zsr} = \frac{p_w}{RT_{wsr}}, \qquad T_{wsr} = T_w\left(\frac{p_w}{p}\right)^{\frac{\kappa-1}{\kappa}}. \tag{34}$$

c) If $p < p_w$ and $\mu_w f_w \neq 0$, then:

$$w_{32} = \sqrt{2RT\,\frac{\kappa_w}{\kappa_w-1}\left[1-\left(\frac{p}{p_w}\right)^{\frac{\kappa_w-1}{\kappa_w}}\right]}, \tag{35}$$

$$\rho_{zsr} = \frac{p}{RT_{wsr}}, \qquad T_{wsr} = T_w\left(\frac{p}{p_w}\right)^{\frac{\kappa_w-1}{\kappa_w}}. \tag{36}$$

Designations used in the above formulas: p – pressure in the cylinder, κ – exponent of the adiabatic curve of the working charge in the cylinder, R – individual gas constant.

5. Conclusions

Most essential components presented in paper relate to:

Taking into accounts of the difference of the temperature between the working charge and the temperature of walls bordering the space of the cylinder and as a consequence for make allowance for of this fact in the heat quantity which surrenders to the exchange in the given time interval.

Working out generalised mathematical model of the working process in the cylinder of the piston four-stroke combustion engine.

At the mathematical modelling processes in the cylinder of the combustion engine, one used dependences describing specific heat of the working charge, which is treated as the gas half-perfect in which the dependence from the temperature is linear.

Worked out mathematical model takes into account nonstationary processes of forming working cycle of the engine realized during openings of valves (open system), the quantity and composition of the working charge in the cylinder and its specific heat and the temperature which are variable. Knowledge of exact values of these magnitudes has a very essential influence on accuracy of results, received both as result of calculations carried out according to proposed mathematical model the cycle, and as a analysis result of the data received from experiment.

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